hexis

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July 30, 2019

1 Introduction

> The general run of current translations of the *Ethics* has settled on the word "state" to render the meaning of *hexis*. But what defines a *hexis* is that it is not a passive state but an active condition, a way in which we hold ourselves, having taken hold deliberately of the feelings and dispositions that are in us merely passively.
>
> (Preface to) *Nicomachean Ethics* [5]
> Joe Sachs

hexis is a programming language.

2 Type judgements

> ...de todos ha de haber en el mundo...
>
> *El Ingenioso Hidalgo Don Quijote de la Mancha* [6]
> Miguel de Cervantes

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2.1 Caveat emptor

THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THE PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

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2.2 Type calculus

People sometimes think that constructive logics (intuitionistic, linear) are weaker, since "they prove less". As remarked by Kreisel long ago, this is a complete mistake: intuitionistic disjunction is not [classical disjunction] with one hand tied behind its back, it is a different operation.

Locus Solum [3]
Jean-Yves Girard

Axioms

Id $\dfrac{}{x : A \vdash x : A}$

Structural rules

Exchange $\dfrac{\Gamma, \Delta \vdash x : A}{\Delta, \Gamma \vdash x : A}$

Cut $\dfrac{\Gamma \vdash x : A \qquad y : A, \Delta \vdash z : B}{\Gamma, \Delta \vdash z[x/y] : B}$

Logical rules

$$\mathbf{1}\mathsf{R}\quad \frac{}{\vdash () : \mathbf{1}} \qquad \mathbf{1}\mathsf{L}\quad \frac{\Gamma \vdash x : A}{\Gamma, y : \mathbf{1} \vdash \mathbf{let}\ () \leftarrow y\ \mathbf{in}\ x : A}$$

$$\otimes\mathsf{R}\quad \frac{\Gamma \vdash x : A \qquad \Delta \vdash y : B}{\Gamma, \Delta \vdash (x, y) : A \otimes B} \qquad \otimes\mathsf{L}\quad \frac{\Gamma, x : A, y : B \vdash z : C}{\Gamma, (x, y) : A \otimes B \vdash z : C}$$

$$\&\mathsf{R}\quad \frac{\Gamma \vdash x : A \qquad \Gamma \vdash y : B}{\Gamma \vdash (x \,\&\, y) : A \,\&\, B} \qquad \&\mathsf{L}_1\quad \frac{\Gamma, x : A \vdash z : C}{\Gamma, w : A \,\&\, B \vdash \mathbf{let}\ (x \,\&\, \_) \leftarrow w\ \mathbf{in}\ z : C}$$

$$\&\mathsf{L}_2\quad \frac{\Gamma, y : B \vdash z : C}{\Gamma, w : A \,\&\, B \vdash \mathbf{let}\ (\_ \,\&\, y) \leftarrow w\ \mathbf{in}\ z : C}$$

$$\oplus\mathsf{R}_1\quad \frac{\Gamma \vdash x : A}{\Gamma \vdash \mathrm{inl}(x) : A \oplus B} \qquad \oplus\mathsf{L}\quad \frac{\Gamma, x : A \vdash z : C \qquad \Gamma, y : B \vdash w : C}{\Gamma, v : A \oplus B \vdash \mathbf{case}\ v\ \mathbf{of}\ \mathrm{inl}(x) \Rightarrow z \mid \mathrm{inr}(y) \Rightarrow w : C}$$

$$\oplus\mathsf{R}_2\quad \frac{\Gamma \vdash y : B}{\Gamma \vdash \mathrm{inr}(y) : A \oplus B}$$

$$\multimap\mathsf{R}\quad \frac{\Gamma, x : A \vdash y : B}{\Gamma \vdash \lambda x.\, y : A \multimap B} \qquad \multimap\mathsf{L}\quad \frac{\Gamma \vdash x : A \qquad y : B, \Delta \vdash z : C}{\Gamma, f : A \multimap B, \Delta \vdash z[fx/y] : C}$$

$$\text{Dereliction}\quad \frac{\Gamma, x : A \vdash y : B}{\Gamma, z : A! \vdash \mathbf{let}\ !x \leftarrow z\ \mathbf{in}\ y : B}$$

$$!\mathsf{R}\quad \frac{!\Gamma \vdash x : A}{!\Gamma \vdash !x : !A} \qquad \text{Weakening}\quad \frac{\Gamma \vdash y : B}{\Gamma, x : !A \vdash \mathbf{let}\ \_ \leftarrow x\ \mathbf{in}\ y : B}$$

$$\text{Contraction}\quad \frac{\Gamma, x : !A, y : !A \vdash w : B}{\Gamma, z : !A \vdash \mathbf{let}\ x \leftarrow z; y \leftarrow z\ \mathbf{in}\ w : B}$$

2.3 Eliminating Cut

This is the rule that expresses the transitivity of implication in sequent calculus, in fact the only really deductive rule of logic. Cut is the possibility to use and reuse lemmas. Cut-elimination is therefore a sort of miracle.

Locus Solum [3]
Jean-Yves Girard

References

[1] S. Abramsky, "Computational interpretations of linear logic," *Theoretical Computer Science*, vol. 111, pp. 3–57, 1993.

[2] P. Wadler, "A taste of linear logic," in *Mathematical Foundations of Computer Science 1993* (A. M. Borzyszkowski and S. Sokołowski, eds.), vol. 711 of *Lecture Notes in Computer Science*, pp. 185–210, Springer-Verlag, 1993.

[3] J.-Y. Girard, "Locus solum: From the rules of logic to the logic of rules," *Mathematical Structures in Computer Science*, vol. 11, no. 3, pp. 301–506, 2001.

[4] Free Software Foundation, "GNU Affero general public license." https://www.gnu.org/licenses/agpl-3.0.en.html, 2007.

[5] J. Sachs, *Nicomachean Ethics*, ch. i (preface). Focus Publishing, 2002.

[6] M. de Cervantes, *El Ingenioso Hidalgo Don Quijote de la Mancha*, ch. 6. Francisco de Robles, 1605.